A Synopsis

of

The Ph.D. Thesis Entitled

"**Some Unsteady Hydromagnetic flow Problems in Channels**"

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Some Unsteady Hydromagnetic flow Problems in Channels

1. Introduction

1.1 Fluid mechanics

Fluid mechanics is the sub-discipline of continuum mechanics that deals with the study of continuous material with no definite shape. Further the fluid dynamics is the sub-discipline of fluid mechanics that deals with the fluid flow-the natural science of fluid in motion. It has several sub-disciplines itself, including aerodynamics (the study of air and gases in motion) and hydrodynamics (the study of liquid in motion). Its study is important to physicists, whose main interest is to understand the phenomenon. They may be interested in learning what cause the various types of wake phenomena in the atmosphere and in the ocean, why a tennis ball hit with 'topspin' dips rather sharply, how fish swim and how birds fly?

1.2 Fluid

Fluid can be defined as, "a substance which deform continuously without limit due to the application of shearing stress, no matter how small it is". This continuous deformation under the action of forces compels the fluid to flow and this tendency of fluid is called 'fluidity'. The flow of the fluid is governed by the following equations:

1.2.1 Equation of State

Variable that depend only upon the state of system are called variable of state. The variables of state are the pressure p, the density ρ and the temperature T. It is an experimental fact that a relationship between the three thermodynamics variable exist and can be written as

$$F(p, \rho, T) = 0$$

1.2.2 Equation of Continuity (Conservation of Mass)

This equation expresses that the rate of generation of mass within a given volume is entirely due to the net inflow of mass through the space enclosing the given volume (assuming that there are no sources). It amounts the basic physical law that the matter is conserved: it is neither being created nor destroyed.

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v_j)}{\partial x_j} = 0$$

where v_j is the j^{th} component of the velocity.

1.2.3 Equation of Motion (Conservation of Momentum)

The equations of motion are derived from Newton's second law of motion which states that

"Rate of change of linear momentum = Total force"

The Navier- Stokes equations describe the motion of fluids such as liquid and gases. These equations establish that change in momentum in infinitesimal volume of fluid are simply the sum of dissipative viscous force, change in pressure, gravity and other forces acting inside the fluids. The Navier- Stokes equations are differential equations which unlike algebraic equations don't explicitly establish a relation among the variable of interest (e.g. velocity and pressure), rather they establish relation among the rates of change. For viscous compressible fluid, Navier-Stokes equations can be expressed as

$$\rho\left[\frac{\partial v_i}{\partial t}+v_j\frac{\partial v_i}{\partial x_j}\right]=\rho X_i-\frac{\partial p}{\partial x_i}+\frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right)-\frac{2}{3}\mu\delta_{ij}\frac{\partial v_k}{\partial x_k}\right]$$

where $X_i=(0,0,-g)$ is the external force, μ is the coefficient of viscosity and δ_{ij} is Kronecker delta. These, based upon a unit of fluid volume, are known as the Navier- Stokes equations.

1.2.4 Equation of Energy (Conservation of Energy)

To obtain the energy equation, we have to apply the law of conservation of energy which states that, the difference in the rate of supply of energy to a controlled surface S enclosing volume V region occupied by a moving fluid and the rate at which the energy goes out through S must be equal to the net rate of increase of energy in the enclosed volume V.

For viscous compressible fluids, the equation of energy is

$$\rho\frac{D}{Dt}(C_pT)=\frac{Dp}{Dt}+\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x_i}\left(k\frac{\partial T}{\partial x_i}\right)+\phi$$

where $\frac{D}{Dt}=\frac{\partial}{\partial t}+v_j\frac{\partial}{\partial x_j}$, is 'material derivative'.

and $\phi=\mu\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}-\frac{2}{3}\delta_{ij}\frac{\partial v_k}{\partial x_k}\right)\frac{\partial v_i}{\partial x_j}$,

is heat transfer due to frictional forces and is usually known as "dissipation function".

1.3 Classification of Fluids

1.3.1 Newtonian and non-Newtonian Fluids

A fluid is said to be Newtonian if its viscosity μ does not change with the rate of deformation. Air, water and glycerin are few examples of Newtonian fluids. In this case $\tau=\mu\frac{du}{dy}$ are similar

to $y = mx$, where, $y = \tau, m = \mu$ and $x = \frac{du}{dy}$. This implies that the shear stress varies linearly with the rate of strain.

A fluid is said to be non-Newtonian if its viscosity varies with the rate of deformation. For such fluids, $\tau, \mu, \frac{du}{dy}$ all varies in equation $\tau = \mu \frac{du}{dy}$. Therefore, non-Newtonian fluid is represented by curve. Blood, grease and sugar are some example of non-Newtonian fluid.

1.3.2 Viscoelastic Fluid

Fluid having both viscous and elastic properties is called viscoelastic fluid. Viscous material like honey, resist shear flow and strain linearity with time when a stress is applied.

1.3.2.1 Constitutive Equation for Walters (Model B)-Viscoelastic Fluid

The constitutive equation for a Walters-B liquid in tensorial form may be presented as follows

$$p_{ik} = -pg_{ik} + p^*_{ik}, \quad p^*_{ik} = 2\int_{-\infty}^{t} \Psi(t - t^*) e^{(1)}_{ik}(t^*) dt^*,$$

p_{ik} is the stress tensor , p is arbitrary isotropic pressure, g_{ik} is the metric tensor of a fixed coordinate systemx_i, $e^{(1)}_{ik}$ is the rate of strain tensor.

$$\Psi(t - t^*) = \int_0^{\infty} \frac{N(\tau)}{\tau} e^{\left(\frac{-(t-t^*)}{\tau}\right)} d\tau$$

$$p^*_{ik}(x, t) = 2\int_{-\infty}^{t} \Psi(t - t^*) \frac{\partial x^i}{\partial x^{*m}} \frac{\partial x^k}{\partial x^{*r}} e^{(1)mr}(x^* t^*) dt^*$$

$N(\tau)$ is the distribution function of relaxation times τ, $x^*_i = x^*_i(x, t, t^*)$ denote the position at time t^* of the element which is instantaneously at the position, x_i , at time t.

For fluids with short memory i.e. low relaxation times, it can be simplified to

$$p^*_{ik}(x, t) = 2\eta_0 e^{(1)k} - 2k_0 \frac{\partial e^{(1)k}}{\partial t}$$

Liquids' obeying $\eta_0 = \int_0^{\infty} N(\tau)\, d\tau$ defines the limiting Walters-B viscosity at low shear rates, $k_0 = \int_0^{\infty} \tau N(\tau)\, d\tau$ is the Walters-B viscoelasticty parameter and $\frac{\partial}{\partial t}$ is the convected time derivative.

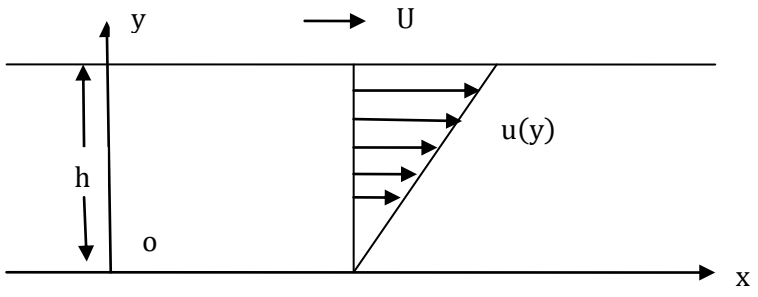
1.4 Types of flow

1.4.1 Steady and Unsteady flow

Steady flow refers to the flow where the fluid properties at a point in the system do not change over time. Otherwise, flow is called unsteady.

1.4.2 Couette flow

In fluid dynamics Couette flow refers to laminar flow of viscous fluid in the space between two parallel plates, one of which is moving relative to other. The flow is driven by virtue of viscous drag force acting on the fluid and applied pressure gradient parallel to the plate. This type of flow is named in honour of Maurice Marie Alfred Couette, a professor of physics of the French University. The simplest conceptual configuration finds two infinite, parallel plates separated by a distance h. One plate, say the top one, translates with a constant velocity U in its own plane. Neglecting pressure gradients, the Navier-Stokes equations are simplified.



Simple Couette configuration using two infinite flat plates.

1.4.3 Rotating Flows

A body of fluid in steady rotation is able to sustain a wave motion propagating along the axis of rotation with axial symmetry. Such rotating flow system exhibits a number of interesting properties which are still being studied actively.

When the motion is referred to an axis which rotates steadily with the basis of the fluid, the Coriolis and centrifugal forces must be supposed to be act on the fluid. The centrifugal force per unit mass may be written as $\frac{1}{2}\nabla(\vec{\Omega}\times\vec{x})^2$ and is equivalent in its effect to a contribution to the pressure in a fluid of uniform density, where $\vec{x}$ is a position vector. The Coriolis force on the other hand give rise to effects of a new type, among which is the elasticity of the fluid. With p' now denoting a modified pressure which includes allowance for the centrifugal force as well as for gravity, the equation of motion with velocity $\vec{v}$ relative to axes rotating with steady angular velocity$\vec{\Omega}$ is

$$\frac{\partial\vec{v}}{\partial t'}+\vec{v}.\nabla\vec{v}+2\vec{\Omega}\times\vec{v}=-\frac{1}{\rho}\nabla p'$$

1.4.4 Chemically reacting flow

In convective heat and mass transfer processes, diffusion rates can be altered tremendously by chemical reaction. The effect of chemical reaction depends on whether the reaction is

heterogeneous or homogenous. In particular, a reaction is said to a first order, if the rate of reaction is directly proportional to the concentration. In nature, the presences of pure water or air are not possible. Some foreign mass either may be present naturally or mixed with air or water. The presences of a foreign mass cause some kind of chemical reaction. The study of such type of chemical reaction processes is useful for improving a number of chemical technologies, such as food processing, polymer production and manufacturing of ceramics or glassware.

1.4.4.1 Effect of chemical reaction

The mass conservation or concentration equation in the presences of a chemical reaction has the following form in three dimensions

$$\frac{\partial C^*}{\partial t^*} + u^* \frac{\partial C^*}{\partial x^*} + v^* \frac{\partial C^*}{\partial y^*} + w^* \frac{\partial C^*}{\partial z^*} = D\left(\frac{\partial^2 C^*}{\partial x^{2*}} + \frac{\partial^2 C}{\partial y^{2*}} + \frac{\partial^2 C^*}{\partial z^{2*}}\right) + m'''$$

Here $m'''$is a chemical reaction term and is finite.

1.4.5 Free convection or Natural convection

Free convection or natural convection is a phenomenon, which occur when a fluid or gas moves because of density changes, which occur inside it due to temperature differences. Heavier (more dense) components will fall while lighter (less dense) components rise, leading to bulk fluid movement. Natural convection can only occur, therefore in a gravitational field. A common example of natural convection is a pot of boiling water in which the hot and less dense water on the bottom layer moves upward in plumes, and the cool and denser water near the top of the pot likewise sink.

1.4.6 Flow through porous medium

A porous medium is a solid with pores in it. 'Pores' are void spaces, which must be distributed more or less frequently throughout the material if it is to be called 'pores'. When a fluid permeates through a porous medium, the actual path of an individual fluid particle cannot be found because of the fluid-rock boundary conditions, which must be considered. Thus in a porous medium one generally considers the fluid motion in terms of volume or ensemble average of the motion of individual fluid elements over regions of space. This was usually done by famous Darcy's[1] Law, as a result of this the viscous term in the equations of fluid motion will be replaced by the resistance term, $-\frac{\mu}{k_1}\vec{q}$ where μ is the viscosity of the fluid, k_1 the permeability of the medium and $\vec{q}$ the seepage velocity of the fluid.

A macroscopic equation which describes incompressible creeping flow of a Newtonian fluid of viscosity μ through a macroscopically homogeneous and isotropic porous medium of permeability k_1 is the well-known **Darcy's equation** $\boldsymbol{-\frac{\mu}{k_1}\vec{q} = \nabla p^*}$,where, p^* is the interstitially average pressure within the porous medium and $\vec{q}$ is the filter velocity (or Darcian velocity).

1.4.6.1 Porosity

The ratio of the volume of the voids (V_v , the space between the solid particles) to the total volume is known as the void age or porosity, which is defined as $\phi = \frac{V_v}{V_v+V_s}$, $V_v = \frac{V_s\phi}{1-\phi}$

1.4.6.2 Permeability

Permeability is that property of a porous material which characterizes the ease with a fluid may be made to flow through the material by an applied pressure gradient. Permeability is the fluid conductivity of the porous material which is defined as $K = \frac{q\mu}{A(\frac{\Delta P}{L})}$.

ΔP is the applied pressure gradient across the length of the specimen. The value of the permeability K is determined by the structure of the porous material.

1.5 Hydromagnetic or Magnetohydrodynamics

Hydromagnetic or Magnetohydrodynamics (MHD) is the science which deals with the motion of electrically conducting fluid in the presence of magnetic fields. It is the synthesis of two classical sciences, Fluid Mechanics and Electromagnetic field theory. It is well known result in electromagnetic theory that when a conductor moves in a magnetic field, electric currents are induced in it. These current experience a mechanical force, called **'Lorentz Force'**, due to the presence of magnetic field. This force tends to modify the initial motion of the conductor. Moreover, induced currents generate their own magnetic field which is added on to the primitive magnetic field. Lorentz force, the force exerted on a charged particle q_1 moving with velocity $\vec{v}$ through an electric field $\vec{E}$ and magnetic field $\vec{B}$. The entire electromagnetic force $\vec{F}$ on the charged particle is called the Lorentz force (after the Dutch Physicist Hendrik A. Lorentz) and is given by

$$\vec{F} = q_1 \vec{E} + q_1(\vec{v} \times \vec{B}).$$

The first term is contributed by the electric field. The second term is the magnetic force and has a direction perpendicular to both the velocity and the magnetic field.

1.5.1 Hall current effect

Hall, a graduate student at John Hopkins University discovered the Hall effect in 1879**.** The tendency of electric current to flow across an electric field in the presence of a magnetic field is called the Hall effect. A comprehensive discussion of hall current is given by Cowling[6] .Taking Hall current into account the generalized Ohm's Law in the absence of electric field is of the form.

$$\vec{J} + \frac{\omega_e \tau_e}{B_0}\left(\vec{J} \times \vec{B}\right) = \sigma\left(\vec{J} \times \vec{B} + \frac{1}{en_e}\nabla p_e + \vec{E}\right)$$

where e, ω_e, τ_e, p_e, n_e and σ are respectively electronic charge , electron frequency, electron collision time, electron pressure, electron number density and electric conductivity are respectively.

1.6 Radiation Effect

Transmission of heat takes place via conduction, convection and radiation. Heat transfer by radiation is explained on the basis of radiant energy that is emitted by the bodies. Thermal radiation is an electromagnetic phenomenon. The radiation between wavelengths of 0.1 to 100 microns is treated as thermal radiation. Electromagnetic radiations, when absorbed by a system, produce thermal energy, that is, a heating effect.

1.7 Soret Effect

If two regions in a mixture are maintained at different temperatures so that there is a flux of heat, it has been found that a concentration gradient is set up. In binary mixtures, one kind of molecules tends to travel towards the hot region and other kind toward the cold region. This is called the **Soret effect**. Soret effect in gas mixtures was first observed and reported by John Tyndall in 1870 and further understood by John Strutt in 1882. In liquid it was first observed and reported by Carl Ludwig in 1865 and further understood by Soret[2]. The equation of mass transfer when Soret effects are included is of the form:

$$\frac{DC}{Dt} = D\nabla^2 C + D_1 \nabla^2 T ,$$

where D is molecular diffusivity and D_1 is thermal diffusivity.

In general, Soret effect is of smaller order of magnitude than the effects described by Fourier's or Fick, law and is often neglected in heat and mass transfer processes. However, exceptions are observed there in. Soret effect, for instances has been utilized for isotope separation, and in mixtures between gases with very light molecular weight (H_2, He) and of medium molecular weight (N_2, air).

1.8 Boundary conditions

The governing equations for the velocity and pressure fields are partial differential equations that are applicable at every point in a fluid that is being modeled as a continuum. When they are integrated in any given situation, we can expect to see arbitrary functions or constants appear in the solution. To evaluate these, we need additional statements about the velocity field and possibly its gradient at the natural boundaries of the flow domain. Such statements are known as *boundary conditions*. Usually, the specification of the pressure at one point in the system suffices to establish the pressure fields so that we shall only discuss boundary conditions on the velocity field here.

1.8.1 No Slip boundary condition

No slip condition means that there is no relative motion between the fluid near surface and solid surface and it requires that

$$\left.\begin{array}{c} u = 0 \text{ at } \quad y = 0 \\ \text{and} \\ u = u_0 \text{ at the moving plate} \end{array}\right\}$$

and it hold good for all fluid except super cooled helium.

1.8.2 Slip Flow Regime

Rarefied gas dynamics is concerned with flows at such low density that the molecular mean free path is not negligible. Under these conditions, the gas no longer behaves as a continuum. However, when the gas is only slightly rarefied, results agreeing with the observed physical phenomena can be obtained by solving the usual the Navier –Stokes equation together with the modified boundary conditions for a velocity and temperature jump at the surfaces. This scheme of theoretical investigation of the so called slip flow regime is particularly suitable for studying the effect of gas rarefaction on any classical viscous flow problem.

It is convenient to subdivide rarefied gas dynamics into four different flow regimes. These are called "free molecular flow", "near-free-molecular flow", "transition flow", "slip flow" corresponding, respectively to extremely rarified, highly rarefied, moderately rarified and only slightly rarefied gas flows. This subdivision is desirable because the four flow regime exhibit quite different phenomenon and the basic theoretical approaches are entirely different. Since rarefied is a relative term, the demarcation of these four subdivisions is not characterized by absolute pressure or gas density level, but rather in terms of the ratio of the mean free path λ to some dimensions L^* characteristics of the flow field. The ratio $\frac{\lambda}{L^*}$ is called Knudsen numberK_n. Free molecular flow corresponds to very large Knudsen number, $K_n \geq 0$; slip flow corresponds to Kundsen number in the range $.01 < K_n < 0.1$; while the transitional regimes lies in between, with. $0.10 < K_n < 10$.

1.9 Origin of Boundary layer Theory

Ludwig Prandtl[3] contributed one of the most important advances in fluid motion mechanics in 1904. He suggested that the fluid motion around objects could be divided into two regions (i) a very thin layer in the neighborhood of the body (known as the boundary layer) in which the viscous effects may be considered to be confined and (ii) the region outside this layer where the viscous effects may be considered as negligible and the fluid is regarded as inviscid. With the aid

of this hypothesis, he simplified the Navier- Stokes equations to a mathematically tractable form, which are then called the boundary layer equations and thus succeeded in giving a physically penetrating explanation of the importance of viscosity in the assessment of frictional drag.

2. Review of the literature

There are in existence several methods which have been developed for the purpose of artificially controlling the boundary layer. The purpose of these methods is to affect the whole flow in a desired direction by in influencing the structure of the boundary layer in order to reduce drag and attain high left. The treatise entitled "Boundary layer and flow control" by G.V. Lachmann[5] contains a summary of the subject of boundary layer control according to the state of research at that time. Several researches have appeared in literature and below we have listed only a few of them, which are relevant to our research work.

Hartmann[4] has studied the theory of the laminar flow of an electrically conducting liquid in a homogenous magnetic field. Ostrach[7] has studied the variable convection in vertical channel with heating from below including effect of heat source and frictional heating. Alpher[8] has investigated the heat transfer in magnetohydrodynamic flow between parallel plates. Nigam and Singh[9] was analyzed the heat transfer by laminar flow between parallel plates under the action of transverse magnetic field. Walter[10] was analyzed the non-Newtonian effects in some elastico-viscous liquids whose behavior at small rate of shear is characterized by a general linear equation of state. Vidyanidhu and Nigam[11] was discussed the secondary flow in a rotating channel. Pop[12]and Mazumder etal[13] has analyzed the effect of Hall currents on hydromagnetic flow near an accelerated plate. Raptis[14] has analyzed the unsteady free convection flow through a porous medium. Jha and Singh[15] has analyzed the Soret effects on free convection and Mass transfer flow in the Stokes problem for an infinite vertical plate. Mazumder [16] was discussed the exact solution of oscillatory Couette flow in a rotating system. Singh and Rana[17] was discussed the three dimensional flow and heat transfer through a porous medium. Attia and Kotb[18] has analyzed the MHD flow between two parallel plate with heat transfer. Raptis[19] has analyzed the radiation and free convection flow through a porous medium. Hall etal[20] was discussed the natural convection cooling of vertical rectangular channels in air considering radiation and wall conduction. Hakiem[21] was discussed MHD oscillating flow on free convection radiation through a porous medium with constant suction velocity. Bakier[22] was discussed the thermal radiations effects on mixed convections from vertical surfaces in saturated porous media. Muthucumaraswamy and Ganesan[23] has analyzed the first order chemical reaction on flow past an impulsively started vertical plate with uniform heat and mass flux. Singh[24] has analyzed the MHD free convection and mass transfer flow with heat sources and thermal diffusion. Attia[25] has studied the influences of temperature dependent viscosity on MHD couette flow of dusty fluid with heat transfer. Sanyal and Adhikari[26] has studied the effect of radiation on MHD vertical channel flow. Hakiem and Rashad[27] has analyzed the effect of radiation on non –Darcy free `convection from a vertical cylinder embedded in a fluid saturated porous medium with a

temperature dependent viscosity. Linga Raju[28] was discussed the Magnetohydrodynamic slip-flow regime in a rotating channel. Muthucumraswamy and Kulandaive[29] has analyzed the radiation effect on moving vertical plate with variable temperature and uniform mass diffusion. Singh and Mathew[30] has analyzed the injection/suction effects on an oscillatory hydromagnetic flow in a rotating horizontal porous channel. Al-Azab[31] was discussed the unsteady mixed convection heat and mass transfer past an infinite porous plate with thermophoresis effect. Rashad[32] was analyzed the perturbation analysis of radiative effect on free convection flows in porous medium in pressure work and viscous dissipation. Dash[33] has studied the effect of chemical reaction on free convection flow through a porous media bounded by vertical surfaces. Singh and Pathak[34] was analyzed the oscillatory free and forced convection flow through a porous medium filled in a rotating vertical channel with slip flow conditions and radiation heat. Srinivas and Muthuraj[35] was analyzed the MHD flow with slip effects and temperature-dependent heat sources in a vertical wavy porous space. Anand Rao and Shivaish[36] has analyzed the chemical reaction effects on an unsteady MHD free convective flow past a vertical porous plate in the presences of a vertical oscillating plate with variable temperature. Chand and Kumar [37] has discussed the Soret and hall current effects on heat and mass transfer in MHD flow of viscoelastic fluid past a porous plate in a rotating porous medium in slip flow regime. Hayat etal[38] has analyzed the radiation effects on MHD flow of Maxwell fluid in a channel with porous medium. Murthya et. al[39] was discussed the effect of chemical reaction on convective heat and mass transfer through a porous medium in a rotating channel. Singh etal[40] was analyzed the effects of thermophoresis on hydromagnetic mixed convection and mass transfer flow past a vertical permeable plate with variable suction and thermal radiation.

3. Theoretical background

The science of fluid dynamics describes the motion of liquids and gases and their interaction with solid bodies. It is a broad, interdisciplinary field that touches almost every aspect of our daily lives, and it is central to much of science and engineering. Fluid dynamics impacts defenses, homeland security, transportation, manufacturing, medicine, biology, energy and the environment. Predicting the flow of blood in the human body, the behavior of micro fluidic devices the aero-dynamics performance of airplanes, cars, and ships, the cooling of electronic components, or the hazard of weather and climate, all require a detailed understanding of fluid dynamics and therefore substantial research. Fluid dynamics is one of the most challenging and exciting fields of scientific activity simply because of the complexity of the subject and the breadth of the applications. The quest for deeper understanding has inspired numerous advances in applied mathematics, computational physics, and experimental techniques.

A central problem is that the governing equations (the Navier –Stokes equation) have no general analytical solution, and computational solutions are challenging. Fluid dynamics is exciting and fruitful today in part because newly available diagnostic methods for experiment and parallel computers for simulations and analysis allow researchers to probe the full complexity of fluid

dynamics in all its rich details. The outcomes from future research will have enormous impact. For instance, they will lead to improved predictions of hurricane landfall and strength by understanding the mechanisms that govern their formation, growth, and interaction with the global weather system.

4. Research methodology

We follow the approach of writing the governing equation of the problem in dimensional form and then non-dimensionlized them by using the known non-dimensional parameters (e.g. Reynolds number, Prandtl number, Schmidt number, Grashoff number, chemical reaction parameter, Peclet number, Hartmann number) These non-dimensional partial differential equations are the reduced to ordinary differential equations with the help of perturbation /variable separable techniques and analytically solved under the relevant boundary conditions. The solutions are numerically evaluated with the help of scientific workplace /Math Cad software and results are expressed graphically and discussed.

4.1 Some important non-dimensional parameters

4.1.1 Reynolds Number (Re)

It is defined as the ratio of inertial force to viscous force i.e.

$$\mathrm{Re} = \frac{\text{inertial force}}{\text{viscous force}} = \frac{\rho u^2/L}{\mu U/L^2} = \frac{\rho UL}{\mu} = \frac{UL}{\nu}$$

where U, L, ρ and μ are some characteristics values of velocity, length, density and viscosity of fluid respectively. It is a parameter for viscosity of fluid.

4.1.2 Prandtl number $(\mathbf{Pr})$

It is defined as the ratio of kinematic viscosity to the thermal diffusivity of the fluid. i.e.

$$\mathrm{Pr} = \frac{\text{kinematic viscosity}}{\text{thermal diffusivity}} = \frac{\nu}{k_0} = \frac{\mu/\rho}{k/\rho C_p} = \frac{\mu C_p}{k}.$$

It is named after the German scientist Ludwig Prandtl. It is a measure of the relative importance of heat conduction and viscosity of fluid.

4.1.3 Grashof number $(\mathbf{G_r})$

The dimensionless quantity G_r, which characterizes the free convection is known as the Grashof number and is defined as

$G_r = \frac{gL^3(T_w - T_\infty)}{\nu^2 T_\infty}$,

where g is the acceleration due to gravity, T_w and T_∞ are two representative temperatures, L is the characteristic length and ν is the kinematic viscosity of fluid.

4.1.4 Hartmann Number (M)

The Hartmann number is obtained from ratio of magnetic field to the viscous force and is defined as

$$M = \sqrt{(\text{Magnetic force})/(\text{Viscous force})} = \mu_e HL\sqrt{\frac{\sigma}{\mu}}$$

where μ_e is the magnetic permeability and H is magnetic field strength and L is characteristic length in the problem.

This number was first used by Hartmann in his classical experiment to channel flow of MHD in which important forces are magnetic force and viscous force.

4.1.5 Nusselt Number (Nu)

It is the dimensionless coefficient of heat transfer. This quantity of heat transfer between the fluid and the surface can be calculated with the help of Newton's law of cooling i.e.

If q(x) is the quantity of heat exchanged between the wall and the fluid, per unit area per unit time, at a point x, then

$q(x) = \alpha(x)(T_w - T_\infty)$, (Newton's law of cooling)

where $(T_w - T_\infty)$ is the difference between the temperature of wall and the fluid. Since at the boundary the heat exchange between the fluid and the body is only due to conduction, according to Fourier's law, we have

$$q(x) = -k\left(\frac{\partial T}{\partial n}\right)_{n=0}$$

where n is the direction of the normal to the surface of the body. From these two laws we can define a dimensionless coefficient of heat transfer which is generally known as the Nusselt Number as follows:

$$Nu = \frac{\alpha(x)L}{k} = -\frac{L}{(T_w - T_\infty)}\left(\frac{\partial T}{\partial n}\right)_{n=0}$$

where L is some characteristic length in the problem.

5. Perturbation technique

Problems in non steady boundary layers involves an steady flow on which there is superimposed a small non steady perturbation. If it is assumed that the perturbation is small compared with the steady basic flow, it is possible to split the equation into non linear equation for the steady perturbation.

The idea is that the problem has a small parameter ϵ in either the governing equation or in the boundary conditions. The solution of these problems can frequently be written in terms of a series involving the small parameter, the higher order term acting as perturbation on the lower order terms. These methods are called perturbation technique. A well known example is that for which the external stream has form

$U(x,t) = \overline{U(x)} + \epsilon U_1(x,t) + \ \ldots\ldots$

where ϵ denote a very small number.

6. Motivation, Aim and Contribution of the proposed Thesis

The above researches and the applications provided a base for the motivation of the present investigations. The aim of the present work is to analyze the flows of fluids in channels. We study the effects of hall current, rotation, magnetic field, Soret, heat source, suction, chemical reaction, radiative heat transfer and slip & jump boundary conditions, on the plane and rotating flows of fluids (Newtonian and non-Newtonian).

The work embodied in the proposed thesis entitled, "**Some Unsteady Hydromagnetic flow Problems in Channels**" is divided into six chapters and represents our attempt to investigate some unsteady and oscillatory flow problems in channels.

7. Plan of the work with work done till date and tentative Chapterization

7.1 Work done till date

Chapter –I Introduction

It is introductory in nature and is devoted to define the basic concept of fluid dynamics and to review the existing literature pertaining to various problems relevant to thesis.

Chapter –II Effect of Hall current and rotation on an oscillatory MHD flow.

The equations governing the unsteady flow of an incompressible, viscous and electrically conducting fluid in a rotating vertical channel in the presence of magnetic field through porous medium used in this chapter are:

Equation of Continuity $\operatorname{div}\vec{V} = 0,$

Momentum Equation: $\rho\left[\frac{\partial\vec{V}}{\partial t} + (\vec{V}\cdot\nabla)\vec{V}\right] + 2\vec{\Omega}\times\vec{V} = -\nabla p + \vec{j}\times\vec{B} + \mu\nabla^2\vec{V} - \frac{\mu}{K}\vec{V} + g\beta T + g\beta C,$

Energy Equation: $\rho C_p\left[\frac{\partial T}{\partial t}+\left(\vec{V}.\nabla\right)T\right]=k\nabla^2T-\nabla q,$

Concentration Equation: $\frac{\partial C}{\partial t}+\left(\vec{V}.\nabla\right)C=D\nabla^2C+D_1\nabla^2T$

Kirchhoff's First Law: $\operatorname{div}\vec{j}=0$

General Ohm's Law: $\vec{j}+\frac{\omega_e\tau_e}{B_0}\left(\vec{j}\times\vec{B}\right)=\sigma\left[\mu_e\vec{V}\times\vec{B}+\frac{1}{e\eta_e}\nabla p_e\right]$

Gauss's Law of Magnetism: $\operatorname{div}\vec{B}=0$

It is divided in two sections.

In section–A above equations has been solved analytically under the assumption of constant suction/injection. The expression obtained for the velocity, the temperature and the concentration field is further used to find other important characteristics of the flow field e.g. the skin friction coefficient, the rate of heat transfer coefficient and mass transfer coefficient.

The result has been numerically evaluated and expressed graphically to depict the effects of various parameters. The results of the study are that the rotation reduces the velocity. Hall current also reduces the velocity for large rotation and enhances the velocity for small rotation parameter.

In section–B the momentum equation in the absence of free convection and mass diffusion i.e. for the horizontal channel has been solved under the assumption of periodic suction/injection at the plate of the channel i.e of the form $\omega=\omega_0(1+\epsilon e^{i\omega t})$, where $\omega_0>0$, $\epsilon \lll 1$ and also the permeability of the porous medium is assumed to be periodic i.e of the form $K=K_0(1+\epsilon e^{i\omega t})$, where $K_0>0$ and $\epsilon \lll 1$.

The solution of the governing equation is obtained by using the complex velocity notation. The result so obtained is used to find the shear stress. The effects of various parameters on the velocity and shear stress have been illustrated with the help of graphs and tables. The results of the study are that the velocity is enhanced by permeability parameter, Hartmann number and rotation and reduces by frequency of oscillation.

Chapter –III Effect of slip and jump conditions on MHD oscillatory flow. This chapter is divided into two sections.
In section –A we have investigated the effect of first order velocity slip and temperature jump boundary conditions at the plate of the channel i.e

$u=\frac{2-f_1}{f_1}\lambda\frac{\partial u}{\partial y}=L_1\frac{\partial u}{\partial y}$, $\quad T-T_0=\frac{2-f_2}{f_1}\frac{2\gamma}{\gamma+1}\frac{\lambda}{Pr}\frac{\partial T}{\partial y}=L_2\frac{\partial T}{\partial y}$.

for hydromagnetic oscillatory Couette flow through porous medium in a vertical porous channel.

In order to discuss the physical significance various parameters on the velocity, the temperature and skin friction at the wall of the channel, real part of the complex results have been considered. Variation in velocity, the temperature and the skin friction have been expressed graphically and discuss. The results of the study are that the velocity profile is enhanced by slip parameter and flow of heat is reversed by the temperature jump.

In section–B we have extended the problem discussed in section –A to find the rarefaction and Darcy effect on the hydromagnetic flow of radiating and reacting fluid in vertical channel.

The governing equations are solved under the relevant boundary conditions. The results have been expressed graphically and it is observed that velocity slip i.e rarefaction effect leads to enhance the fluid velocity and radiation effect reduces the skin friction.

Chapter–IV Hydromagnetic oscillatory flow of dusty fluid. The chapter is divided into three sections. The flow of a dusty fluid used in this chapter in rotating frame of reference is governed by the following Momentum equations

$$\frac{\partial\vec{u}}{\partial t}+(\vec{u}.\nabla)\vec{u}+2\vec{\Omega}\times\vec{u}=\frac{-1}{\rho}\nabla p+\mu\nabla^2\vec{u}+\frac{KN}{\rho}\left(\overrightarrow{u_p}-\vec{u}\right)+\frac{1}{\rho}\vec{J}\times\vec{B}$$

The motion of the dust particles is governed by second law of Newton's and is given by

$$m_p\left[\frac{\partial\overrightarrow{u_p}}{\partial t}+\left(\overrightarrow{u_p}.\nabla\right)\overrightarrow{u_p}+2\Omega\times\overrightarrow{u_p}\right]=K'(\vec{u}-\overrightarrow{u_p})$$

The energy equations describing the temperature distributions for the fluid and dust particles neglecting the viscous dissipation and the Joule dissipation are given by

$$\rho c_v\frac{\partial T}{\partial t}+\rho c_v\omega_0\frac{\partial T}{\partial z}=\kappa\frac{\partial^2 T}{\partial z^2}+\frac{\rho_p c_s}{\gamma_T}(T_p-T)\quad\text{and}\quad\frac{\partial T_p}{\partial t}=-\frac{1}{\gamma_T}\left(T_p-T\right)\text{ respectively}$$

In section–A an analysis of unsteady flow of dusty viscous incompressible and electrically conducting fluid in a horizontal porous channel rotating with constant angular velocity under the conditions of constant suction in the presences of transverse applied magnetic field is studied. Analytical solutions for the velocity of fluid and dust particles are obtained. The results have been numerically evaluated and shown graphically and discussed. The results of the study are that fluid and dust particles velocity are enhanced by the amplitude of pressure gradient and reduces by the rotation parameter and maximum along the centre of the channel.

In section–B an analysis of hydromagnetic unsteady flow of dusty viscous incompressible and electrically conducting fluid in a vertical porous channel rotating with an angular velocity under the influence of periodic pressure gradient is studied.

Analytically solutions for the velocity and temperature (in the absence of viscous dissipation and Joule heating) of fluid and dust particles are obtained and evaluated numerically and expressed

graphically. The result of the study is that the free convection reduces the fluid and dust particle velocity.

In section –C effect of hall current and rotation on heat transfer in MHD flow of dusty viscous incompressible and electrically conducting, oscillatory dusty fluid in porous horizontal is analyzed.

The effect of various parameters on the velocity and temperature profile of both fluid and dust particles phase are shown graphically and discussed. The results of the study are that the rotation and hall current parameter reduce the fluid and dust particle velocity.

Chapter–V Hydromagnetic oscillatory flow through a porous medium bounded by two vertical porous plates with heat source and Soret effect. In this chapter the hydromagnetic oscillatory flow of electrically conducting, viscous incompressible fluid through a saturated porous medium bounded by two insulating vertical parallel porous plates is analyzed by considering the heat source and Soret effects.

Closed form solutions of the governing equations are obtained for the velocity, the temperature, the concentration profiles, the skin friction, the rate of heat and mass transfer coefficient. The results so obtained are numerically evaluated and discussed with the help of graphs and tables. The result of the study is that for species with large diffusion ratio, the velocity is enhanced and skin friction is reduced.

Chapter–VI Hydromagnetic oscillatory flow of viscoelastic fluid. The chapter is divided in two sections. The governing equation of incompressible, electrically conducting flow of viscoelastic fluid through a vertical channel used in this chapter under usual Boussinesq approximation is given by

$$\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho}\frac{\partial P^*}{\partial x^*} - \frac{\sigma B_0^2}{\rho}u^* + \frac{1}{\rho}\frac{\partial \tau^*_{x^*y^*}}{\partial y^{*2}} + g\beta T^* + g\beta_c C^*$$

where $\tau^*_{x^*y^*}$ are the component of shear stress of the viscoelastic fluid.

$$\tau^*_{x^*y^*} = \mu\frac{\partial u^*}{\partial y^*} - \frac{\mu}{\alpha}\frac{\partial \tau^*_{x^*y^*}}{\partial t^*}$$

where μ is the coefficient of viscosity and α is the modulus of rigidity.

In this section–A we theoretically investigate the effect of chemical reaction and radiation on heat and mass transfer of a general unsteady hydromagnetic, free convective flow of viscoelastic fluid (Walter's B-model) through a vertical channel.

The solution of the governing equation for the velocity, the temperature and the concentration fields are obtained. The significant effects of various parameters entering into the problem, on the velocity, the temperature, the concentration, the skin friction, the rate of heat and mass

transfer have been evaluated numerically and expressed graphically. The results of the study are that the amplitude of the velocity field is significantly enhanced by the free convection parameter, the chemical reaction parameter and the amplitude of the pressure gradient whereas the radiation parameter diminishes its amplitude. The amplitude of skin friction coefficient is reduced with the increasing radiation parameters, magnetic field parameter and viscoelastic parameter.

7.2 Work still to be done

In section–B the theoretical investigation of the effect of slip and jump boundary conditions on heat and mass transfer of general unsteady hydromagnetic, free convective flow of chemically reacting and radiating viscoelastic fluid (Walter's B-model) through a porous medium in porous vertical channel is intended to be discussed.

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List of publications

1. Heat and mass transfer in an unsteady MHD convective flow through a porous medium bounded by vertical porous channel in the presence of radiative heat and hall current. International Journal of Mathematical Sciences and Engineering Applications, Vol.6, No. I, 317-336, January 2012, K.D. Singh, Khem Chand and Shavnam Sharma.

2. Hydromagnetic oscillatory vertical Couette flow of radiating fluid through porous medium with slip and jump conditions. International journal of Physical and Mathematical Sciences, Vol-3, No-1, 82-90, March-2012, Khem Chand, Rakesh Kumar and Shavnam Sharma.

3. Hydromagnetic oscillatory flow of dusty fluid in a rotating porous channel, Int. journal of Mathematical Archive, 3(6), 2331-2339, June-2012, K.D. Singh, Khem Chand and Shavnam Sharma.

4. Hydromagnetic oscillatory flow through a porous medium bounded by two vertical porous plates with heat source and soret effect, Advances in Applied sciences and researches, 3(4) 2169-2178, july-2012, Khem Chand , Rakesh Kumar and Shavnam Sharma.

5. Hydromagnetic oscillatory flow through a porous channel in the presence of hall current with variable suction and permeability, International Journal of Statistika and Mathematika Vol. 3(1), 70-76,September-2012, Khem Chand and Shavnam Sharma.

6. Combined effect of chemical reaction and radiation on heat and mass transfer in oscillatory MHD flow of viscoelastic fluid through vertical channel, Research J. Science and Tech. 5(1), 77-84 Jan.-Mar.2013, Khem Chand, K.D. Singh, and Shavnam Sharma.

Paper Accepted for Publication

1. Rarefaction and Darcy effects on the Hydromagnetic flow of radiating and reacting fluid in a vertical channel communicated for publication, accepted for publication in Turkish journal of engineering and environmental sciences**.** Khem Chand, Rakesh Kumar and Shavnam Sharma.

Paper Communicated for Publication

1. Hydrodynamic oscillatory flow of dusty fluid in a rotating porous vertical channel - K.D. Singh, Khem Chand and Shavnam Sharma.

2. Effect of rotation and Hall current on heat transfer in MHD flow of oscillating dusty fluid in a porous channel- Khem Chand, K.D. Singh, and Shavnam Sharma

Paper Under preparation

1. Effect of Slip and Jump boundary conditions on Chemical Reacting and Radiating MHD Free Convective flow of Viscoelastic fluid through a porous medium in porous vertical channel.

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